18-819F: Introduction to Quantum Computing
47-779/785: Quantum Integer Programming & Quantum Machine Learning

First Look at Quantum Algorithms

Lecture 13

2022.10.24

Agenda

- What does computation mean in quantum circuits
  - Quantum parallelism – is it real?
- Deutsch's problem for 1 qubit
  - Deutsch's circuit (algorithm)
  - Analysis of the Deutsch circuit
- Extension of the Deutsch algorithm to n-qubits: the Deutsch-Josza algorithm
  - Analysis of the Deutsch-Josza circuit

Quantum Computational Process

- Simply stated, in a computational process, we want a quantum computer to take a number x and produce another number $f(x)$ by way of some function f; we will think of f as applying a unitary transformation, U_f. Furthermore, U_f is reversible (in that it is its own inverse).

- We assume there is an input register with n qubits and an output register with m qubits in the computer.

- The action of the operator U_f on the computational basis states $|x\rangle_n|y\rangle_m$ of the input and output registers will be defined by

$$U_f(|x\rangle_n|y\rangle_m) = |x\rangle_n|y \oplus f(x)\rangle_m \quad \text{Eqn. (13.1).}$$

- The symbol $\oplus$ is addition modulo-2 and is equivalent to the exclusive OR operation we have already discussed. To illustrate its application, suppose in (13.1) the output register is $y = 0$, then (13.1) reduces to

$$U_f(|x\rangle_n|0\rangle_m) = |x\rangle_n|f(x)\rangle_m \quad \text{Eqn. (13.2).}$$

Quantum Computational Process

- To demonstrate the invertibility of U_f, we operate with it in (13.1) twice as follows

$$U_f U_f(|x\rangle|y\rangle) = U_f(|x\rangle|y \oplus f(x)\rangle) = |x\rangle|y \oplus f(x) \oplus f(x)\rangle = |x\rangle|y\rangle \quad \text{Eqn. (13.3).}$$

- Note that $f(x) \oplus f(x) = 0$ by the definition of the exclusive OR operator we discussed in Lecture 10 (see the truth table for the exclusive OR operator in that Lecture). Alternatively, adding the same bit twice and dividing by 2 leaves a remainder of xero.

- The Hadamard is one of the most important operators in quantum computing; it can be applied to 2-qubit and n-qubit states as follows

$$(H \otimes H)(|0\rangle \otimes |0\rangle) = (H|0\rangle)(H|0\rangle) = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) = \frac{1}{2}(|00\rangle + |01\rangle + |10\rangle + |11\rangle) \quad \text{Eqn. (13.4).}$$

- For an n-qubit state (11.4) generalizes to

$$H^{\otimes n}|0\rangle_n = \frac{1}{2^{n/2}} \sum_{0 \le x < 2^n} |x\rangle_n \quad \text{Eqn. (13.5).}$$

Quantum Parallelism

- From (13.4) and (13.5) we see that the Hadamard produces a superposition of the 2- or n-qubit input and output registers. If we then apply the unitary operator U_f, we see that the final state contains many evaluations of the function f at once. This is called quantum parallelism; it doesn't mean we have access to all the results of the evaluation. Measurement by the Born rule allows us to have only the values that collapse to the measurement basis.

- Application of U_f after H proceeds as follows

$$U_f\left(H^{\otimes n} \otimes 1_m\right)(|0\rangle_n|0\rangle_m) = \frac{1}{2^{n/2}}\sum_{0\leq x<2^n} U_f(|x\rangle_n|0\rangle_m) \quad \text{Eqn. (13.6).}$$

- If we apply 20 Hadamard gates to the input before application of the operator U_f, then in theory the output will contain 2^{20} or over a million evaluations of the function f. These evaluations characterize the state of the output of the computation; measurement causes a collapse into the measurement basis.

Deutsch's Problem

- This problem is about an unknown function f whose inputs are defined in $\{0,1\}$ and outputs in $\{0,1\}$. The question of interest is whether f is *balanced* or *constant*. Balanced means $f(0) \neq f(1)$ and constant means $f(0) = f(1)$.

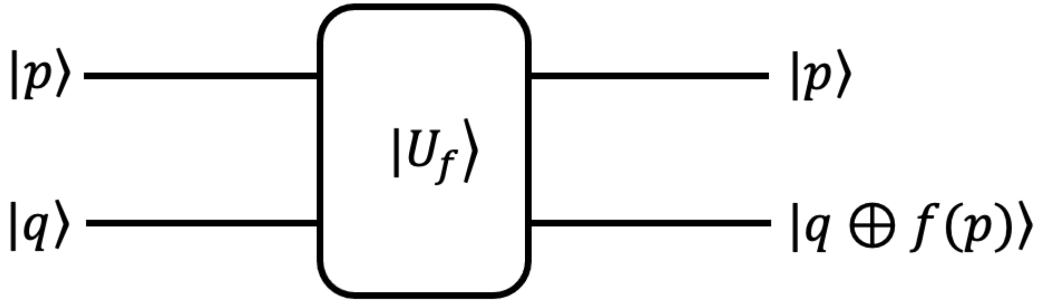
- The classical way to answer the question is to evaluate f for the input 0 and input 1 and then check to see if $f(0) = f(1)$. At the minimum, one requires at least 2 evaluations: one for $f(0)$ and another for $f(1)$ to be able to give an answer. Deutsch wanted to know whether a quantum approach to the computation could answer the question more efficiently (fewer steps). In another words, with just fewer queries than the classical approach.

- This is an optimization problem, where the "cost function" is the number of queries to the operator U_f.

- When the problem is framed this way, we are using the *quantum query complexity* model. In this model, there is a box U_f and the interest is in how many times one must query the box to get a desired answer.

Definition of the Deutsch Problem

- We want an oracle (the box) U_f that can be questioned to determine if function f is constant or balanced. The truth table for the function is shown alongside the box representation of the oracle.



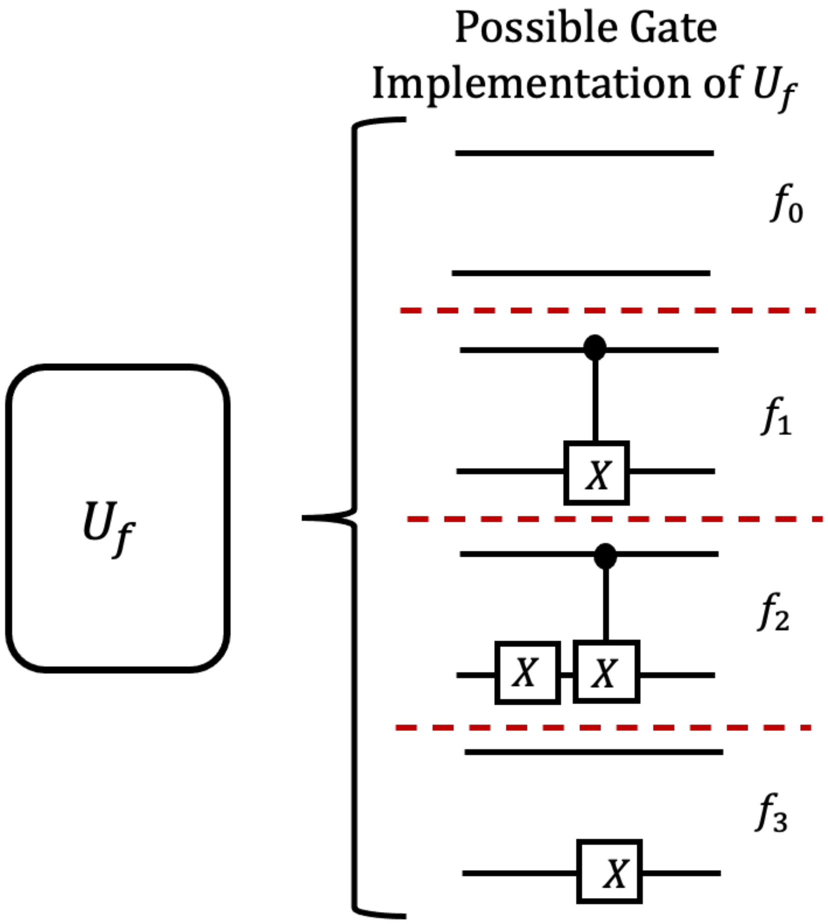
f	$f(0)$	$f(1)$
f_0	0	0
f_1	0	1
f_2	1	0
f_3	1	1

- The oracle performs the general computation given by the expression below.

$$U_f(|p\rangle|q\rangle) = |p\rangle|q \oplus f(p)\rangle$$

Possible Gate Circuit Implementations

- There are four distinct possible values the function can be.
- If the input is $|0\rangle \otimes |0\rangle$ the output is $f_0 = |0\rangle|f(0)\rangle$;
- If the input is $|0\rangle \otimes |1\rangle$ the output is $f_1 = |0\rangle|1 \oplus f(0)\rangle$;
- If the input is $|1\rangle \otimes |0\rangle$ the output is $f_2 = |1\rangle|0 + f(1)\rangle$;
- If the input is $|1\rangle \otimes |1\rangle$ the output is $f_3 = |1\rangle|1 \oplus f(1)\rangle$;
- Different gates as illustrated could achieve the output results. But we want a single circuit. As we will see Deutsch achieved this goal by using the Hadamard gate several times in his circuit.



First Trial Solution to Deutsch's Problem

- A quantum mechanical way to approach the problem is to recall (13.1). However, instead of f having just one qubit in the input register, we provide the input in superposition of the possible inputs {0,1}; We replace $|x\rangle$ in (13.1) with $\alpha|0\rangle + \beta|1\rangle$ and assume the output $|y\rangle$ is $|0\rangle$. When we operate with U_f, we expect the output to contain $f(0)$ and $f(1)$. Thus
  $|\psi\rangle_{out} = U_f(\alpha|0\rangle + \beta|1\rangle) \otimes |0\rangle = \alpha U_f(|00\rangle) + \beta U_f(|10\rangle) = \alpha|0\rangle|f(0)\rangle + \beta|1\rangle\,|f(1)\rangle$ Eqn. (13.7).

- Note that we have used the fact that because $U_f|00\rangle = |0\rangle|0 \oplus f(0)\rangle$ we can write $0 \oplus f(0) = f(0)$, and for $U_f(|10\rangle) = |1\rangle|0 \oplus f(1)\rangle$ we can use $0 \oplus f(1) = f(1)$ to arrive at the last part of (11.7).

- Measurement in the $|0\rangle$ or $|1\rangle$ basis collapses the state into that one of these bases and we still won't have an answer to the original problem of whether f is constant or balanced.

A Preliminary to Deutsch's Algorithm

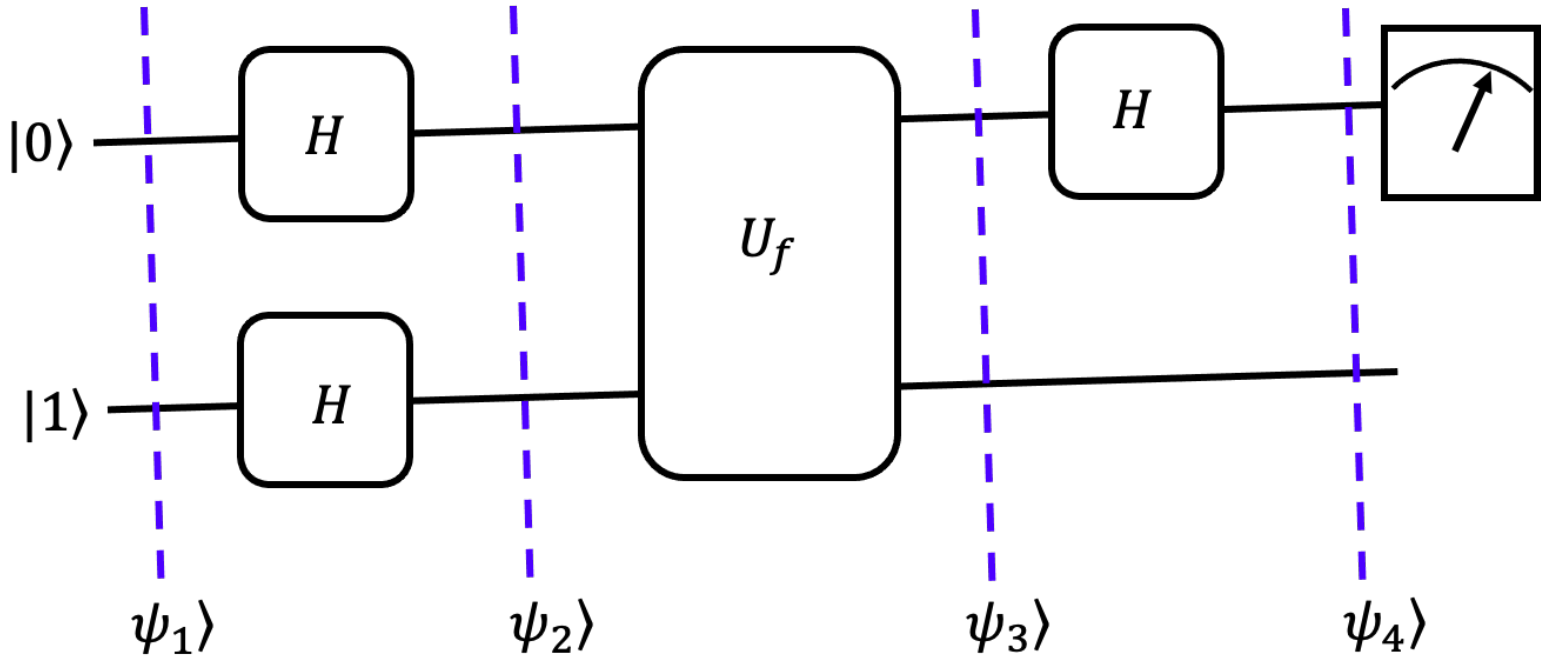
- The goal of Deutsch's problem is to determined if f is constant or balanced: meaning that $f(0) = f(1)$ is constant, and $f(0) \neq (f1)$ is balanced. The goal is equivalent to evaluating $f(0) \oplus f(1)$.

- The constancy of f is proven when we evaluate that $f(0) \oplus f(1) = 0$. The only way the result is zero is when both $f(0)$ and $f(1)$ evaluate to the same bit such that addition the same bit twice and dividing by 2 leaves a remainder of zero.

- The function f is balanced when $f(0) \oplus f(1) = 1$. This result is obtained when the evaluation of $f(0)$ leads to a different bit from evaluation of $f(1)$. When two different bits are added and divided by 2, a remainder is left as indicated.

- These two arguments provide the evidence that what Deutsch set out to do is to evaluate $f(0) \oplus f(1)$.

Circuit for Deutsch's Algorithm



- The circuit that implements Deutsch's algorithm is shown above. We explain how it works in the following slides.

Analysis of the Deutsch Circuit

- The state at the input of the circuit is given by

$$|\psi_1\rangle = |01\rangle \quad \text{Eqn. (13.7).}$$

- After the Hadamard, the state of the system is given as

$$|\psi_2\rangle = |+\rangle|-\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \otimes \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle) = \frac{1}{2}(|0\rangle|0\rangle - |0\rangle|1\rangle + |1\rangle|0\rangle - |1\rangle|1\rangle) \quad \text{Eqn. (13.8)}$$

- After applying the U_f operator the function is given by

$$|\psi_3\rangle = U_f|\psi_2\rangle = \frac{1}{2}\big(|0\rangle|f(0)\rangle - |0\rangle|1 \oplus f(0)\rangle + |1\rangle|f(1)\rangle - |1\rangle|1 \oplus f(1)\rangle\big) \quad \text{Eqn. (13.9).}$$

- If the function f is constant, then we have $f(0) = f(1)$, which allows us to simplify (13.9) to

- $$\left.\begin{aligned} |\psi_3\rangle &= \frac{1}{2}\big(|0\rangle|f(0)\rangle - |0\rangle|1 \oplus f(0)\rangle + |1\rangle|f(0)\rangle - |1\rangle|1 \oplus f(0)\rangle\big) \\ &= \frac{1}{2}[(|0\rangle + |1\rangle) \otimes |f(0)\rangle - (|0\rangle + |1\rangle) \otimes |1 \oplus f(0)\rangle] \\ &= \frac{1}{2}(|0\rangle + |1\rangle) \otimes |f(0)\rangle - |1 \oplus f(0)\rangle \\ &= \frac{1}{\sqrt{2}}|+\rangle \otimes (|f(0)\rangle - |1 \oplus f(0)\rangle) \end{aligned}\right\} \text{Eqn. (13.10).}$$

...Analysis of the Deutsch Circuit

- From (13.10), we see that the first qubit has been transformed to the state $|+\rangle$.
- After the Hadamard, then the system state will be

$$|\psi_4\rangle = H\left|\psi_3\right\rangle = H\left(\frac{1}{\sqrt{2}}|+\rangle\right) \otimes (|f(0)\rangle - |1 \oplus f(0)\rangle) = \frac{1}{\sqrt{2}}|0\rangle \otimes (f(0)\rangle - |1 \oplus f(0)\rangle) \quad \text{Eqn. (13.11)}$$

- If we now measure the first qubit of (13.11) in the standard basis, the state will collapse to 0.
- If the function f is balanced, then $f(0) \neq f(1)$ and $f(0) \oplus 1 = f(1)$ and $f(1) \oplus 1 = \mathrm{f}(0)$, then (13.9) can be simplified to

- $$\left.\begin{aligned} |\psi_3\rangle &= \frac{1}{2}\big(|0\rangle|f(0)\rangle - |0\rangle|f(1)\rangle + |1\rangle|f(1)\rangle - |1\rangle|f(0)\rangle\big) \\ &= \frac{1}{2}\big((|0\rangle - |1\rangle) \otimes |f(0)\rangle - (|0\rangle - |1\rangle) \otimes |f(1)\rangle\big) \\ &= \frac{1}{2}(|0\rangle - |1\rangle) \otimes (|f(0)\rangle - |f(1)\rangle) \\ &= \frac{1}{\sqrt{2}}|-\rangle \otimes \big(|f(0) - |f1)\rangle\rangle\big) \end{aligned}\right\} \text{Eqn. (13.12)}$$

Analysis of the Deutsch Circuit

- From (13.12), we see that the first qubit has been transformed to the state $|-\rangle$.
- After passing through the Hadamard, it is clear that

$$|\psi_4\rangle = \frac{1}{\sqrt{2}}|1\rangle \otimes (|f(0)\rangle - |f(1)\rangle) \quad \text{Eqn.}(13.13)$$

- We now see that the first qubit has been transformed to 1. When that is followed by measurement in the standard basis, we are assured that we will get 1.

- From (13.1) and (13.13) after measurement with a standard basis the circuit outputs, respectively, a 0 when f is constant and a 1 when f is balanced.

- The crucial point that Deutsch realized is that his algorithm can decide with just one query whether f is constant or balanced.

Utilizing a Phase Insight to Simplify Analysis

- The state vector $|+\rangle$ and $|-\rangle$ differ by 180^o phase (indicated by the minus sign).

- We investigate the action of U_f on an input state that has $|-\rangle$ as a component, for example,

- $$\left.\begin{aligned} U_f|x\rangle|-\rangle &= \frac{1}{\sqrt{2}}\big(U_f|x\rangle|0\rangle - U_f|x\rangle|1\rangle\big) \\ &= \frac{1}{\sqrt{2}}(|x\rangle|f(x)\rangle - |x\rangle|1 \oplus f(x)\rangle) \\ &= \frac{1}{\sqrt{2}}|x\rangle \otimes (|f(x)\rangle - |1 \oplus f(x)\rangle) \end{aligned}\right\} \text{Eqn. (13.14).}$$

- It is possible that $f(x) = 0$ or $f(x) = 1$; if $f(x) = 0$ then the last result in (13.14) becomes

$$U_f|x\rangle|-\rangle = |x\rangle \otimes \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle) = |x\rangle|-\rangle \quad \text{Eqn. (13.15)}$$

- And if $f(x) = 1$ then the last result in (11.14) becomes

$$U_f|x\rangle|-\rangle = |x\rangle \otimes \frac{1}{\sqrt{2}}(|1\rangle - |0\rangle) = -|x\rangle|-\rangle \quad \text{Eqn. (13.16).}$$

- We can combine (13.15) and (13.16) and write $U_f|x\rangle|-\rangle = (-1)^{f(x)}|x\rangle|-\rangle$ Eqn. (13.17).

Re-examining the Deutsch Circuit with Phase Insight

- The state vector of the circuit after the Hadamard can be re-written as

  $|\psi_2\rangle = |+\rangle|-\rangle = \frac{1}{\sqrt{2}}(|0\rangle|-\rangle + |1\rangle)|-\rangle)$, Eqn. (13.18), where have not expanded $|-\rangle$.

- With the phase insight, the state vector, $|\psi_3\rangle$, after application of U_f becomes (using (13.17)

  $$|\psi_3\rangle = \frac{1}{\sqrt{2}}\left((-1)^{f(0)}|0\rangle|-\rangle + (-1)^{f(1)}|1\rangle|-\rangle\right) \text{ Eqn. (13.19)}$$

- For f constant, $f(0) = f(1)$; we can therefore factor out $(-1)^{f(0)}$ and rewrite (13.19) as

  $$|\psi_3\rangle = (-1)^{f(0)}\frac{1}{\sqrt{2}}(|0\rangle|-\rangle + |1\rangle|-\rangle) = (-1)^{f(0)}|+\rangle|-\rangle \text{ Eqn.. (13.20)}$$

- When we apply the Hadamard the state vector $|\psi_4\rangle$ becomes

  $$|\psi_4\rangle = (-1)^{f(0)}|0\rangle|-\rangle \text{ Eqn. (13.21).}$$

- Measuring in the standard basis then yields 0 for the first qubit as before.
- Finally, when f is balanced, $f(0) \neq f(1)$, and we cannot factor out the -1; we must now write $|\psi_3\rangle$ as

  $$|\psi_3\rangle = \pm\frac{1}{\sqrt{2}}(|0\rangle|-\rangle - |1\rangle|-\rangle) = \pm|-\rangle|-\rangle \text{ Eqn. (13.22).}$$

Re-examining the Deutsch Circuit with Phase Insight

- Application of the last Hadamard to (13.22) gives us the expression for $|\psi_4\rangle$ as

$$|\psi_4\rangle = \pm|1\rangle|-\rangle \quad \text{Eqn. (13.23).}$$

- Measurement in the standard basis gives us the first qubit as 1 with certainty. This is the same result we obtained earlier.

- Deutsch's problem was posed to determine if quantum computers could do some things more efficiently than classical computers.

- The question then was to find what is meant by more efficiently. It turns out, time of execution is the relevant parameter, but not the only one.

- What Deutsch set out to prove was the *query complexity* aspect of computing. In this case the quantum processor performed better; in some problems this advantage translates to a "faster" execution time.

Deutsch-Josza Algorithm

- Single qubit algorithms are not useful for anything interesting. An enhancement of the Deutsch algorithm that deals with multiple bit inputs is known as the Deutsch-Josza algorithm. It is designed to determine the properties of functions that act in the manner $f: \{0,1\}^n \rightarrow \{0,1\}$;

- A typical problem for the Deutsch-Josza algorithm is to determine the if the $n-$ qubit function f: $\{0,1\}^n \rightarrow \{0,1\}$ is constant or balanced. By constant is meant whether $f(x)$ is the same for all $x \in \{0,1\}$, and by balanced one means that $f(x) = 0$ for half of the inputs $x \in \{0,1\}^n$ and $f(x) = 1$ for the other remaining inputs.

- We will define the oracle (black box) U_f implementing the function f as $U_f|x\rangle|y\rangle = |x\rangle|y \oplus f(x)\rangle$. This time x is an n-qubit string.

- The quantum circuit implementing the Deutsch-Josza algorithm is shown on the next slide.

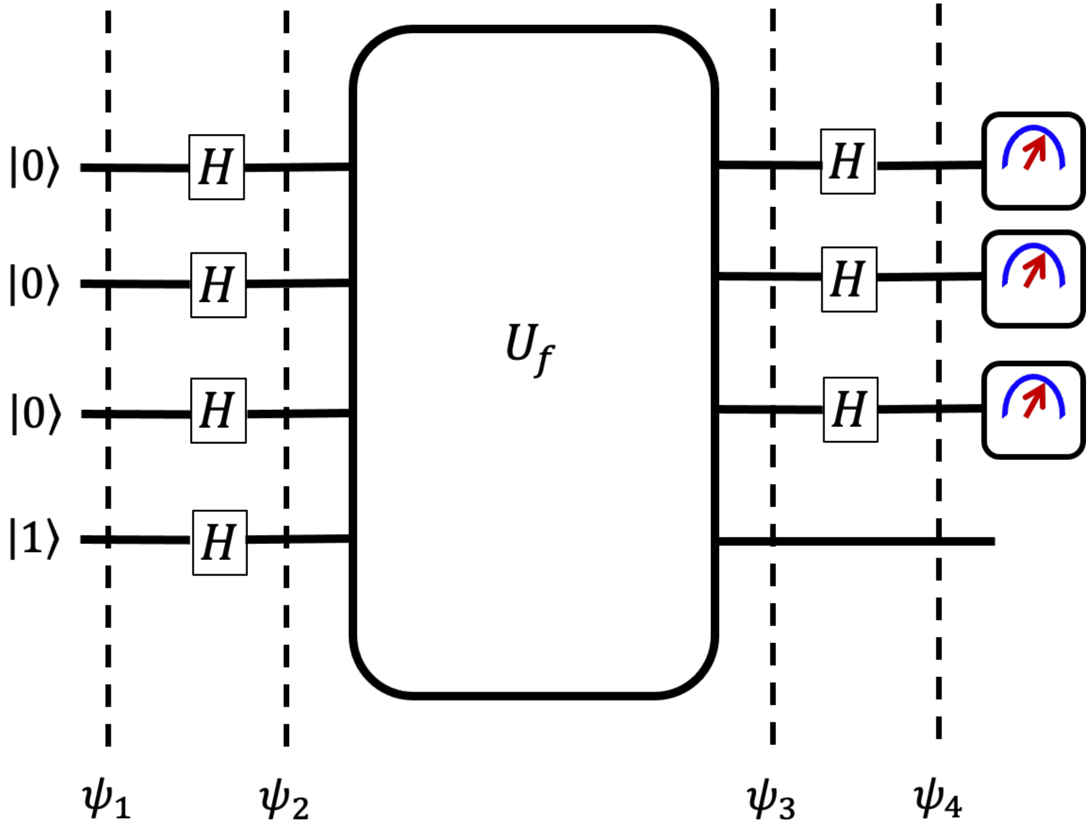
Circuit implementation of the Deutsch-Josza Algorithm

- This circuit is a generalization of the single bit Deutsch algorithm we already discussed.

- We divide the computational process into 4 stages as indicated with the state functions ψ_1, ψ_2, ψ_3 and ψ_4.

- Note that the action of U_f on f is defined in terms of standard basis:

$$U_f|x\rangle|y\rangle = |x\rangle|y \oplus f(x)\rangle$$



State Functions After Each Stage of the Circuit

- Beginning at the input of the circuit, we note that the state can be written as

$$|\psi_1\rangle = |0\rangle \ldots |0\rangle\, |1\rangle = |0\rangle^{\otimes n}|1\rangle \text{ Eqn. (13.23)}$$

- After the Hadamard gates, we have a new state, $|\psi_2\rangle$, given by the action of the Hadamard and the entanglement operator to yield

$$|\psi_2\rangle = H\,|0\rangle \ldots \otimes H|0\rangle \otimes H|1\rangle = |+\rangle \ldots |+\rangle\, |-\rangle = |+\rangle^{\otimes n} H|1\rangle = |+\rangle^{\otimes n}|-\rangle \text{ Eqn. (13.24)}$$

- To determine the state after the U_f operator requires that we rewrite the $|+\rangle^{\otimes n}$ in terms of the standard basis; thus

$$|+\rangle^{\otimes n} = \frac{1}{\sqrt{2}^n}(|0\rangle + |1\rangle) \otimes \cdots \otimes (|0\rangle + |1\rangle) = \frac{1}{2^{n/2}} \sum_{x\in\{0,1\}^n} |x\rangle$$

- In view of the above, we can now write $|\psi_2\rangle$ as

$$|\psi_2\rangle = |+\rangle^{\otimes n}|-\rangle = \frac{1}{2^{n/2}} \sum_{x\in\{0,1\}^n} |x\rangle\,|-\rangle \text{ Eqn. (13.25)}$$

Action of the U_f Operator and the Hadamard

- Since stage 3 is after the U_f operator, we can use the phase kickback trick of $(-1)^{f(x)}$ to write the state $|\psi_3\rangle$ as

$$|\psi_3\rangle = \frac{1}{2^{n/2}} \sum_{x \in \{0,1\}^n} (-1)^{f(x)} |x\rangle\, |-\rangle \quad \text{Eqn. (13.26)}$$

- Applying the last set of Hadamard gates to $|\psi_3\rangle$ leads to the next state $|\psi_4\rangle$, which we write as

$$|\psi_4\rangle = \frac{1}{2^{n/2}} \sum_{x \in \{0,1\}^n} (-1)^{f(x)} H^{\otimes n} |x\rangle\, |-\rangle \quad \text{Eqn. (13.27)}$$

- Our next task is to figure out what exactly is meant by $H^{\otimes n}|x\rangle$ in the expression above for arbitrary $x \in \{0,1\}$.

Action of the Hadamard Operator after the U_f

- Because $H|0\rangle = |+\rangle$ $H|1\rangle = |-\rangle$, for $x_1 \in \{0,1\}$, we can write

$$H|x_1\rangle = \frac{1}{2} \sum_{z_1 \in \{0,1\}} (-1)^{x_1 z_1} |z_1\rangle \quad \text{Eqn. (13.28)}$$

- In general, we can now write

$$H^{\otimes n}|x\rangle = H|x_1\rangle \otimes \cdots \otimes H|x_n\rangle = \frac{1}{2^{n/2}} \sum_{z_1 \in \{0,1\}} (-1)^{x_1 z_1} |z_1\rangle \otimes \cdots \otimes \sum_{z_n \in \{0,1\}} (-1)^{x_n z_n} |z_n\rangle \quad \text{Eqn. (13.29)}$$

- The expression above can be simplified to

$$H^{\otimes n}|x\rangle = \frac{1}{2^{n/2}} \sum_{z_1 \in \{0,1\}^n} (-1)^{x_1 z_1 + \cdots x_n z_n} |z\rangle \quad \text{Eqn. (13.30)}$$

Action of the Hadamard Operator after the U_f

- We note in passing that $x_1 z_1 + \cdots x_n z_n = x.z$ is essential the dot product, which is the power of the (-1) , which we now write as $(-1)^{x.z}$. We only care when the exponent is even or odd. This is the only time when we get -1 for an odd, or 1 for a even exponent.

- Combining this fact and the ones from the previous slide in Eqn. (13.30) allows us to write state $|\psi_4\rangle$ as

$$\left.\begin{aligned} |\psi_4\rangle &= \frac{1}{2^{n/2}} \sum_{x\in\{0,1\}^n} (-1)^{f(x)} H^{\otimes n} |x\rangle \, |-\rangle \\ &= \frac{1}{2^{n/2}} \sum_{x\in\{0,1\}^n} (-1)^{f(x)} \left(\frac{1}{2^{n/2}} \sum_{z_1\in\{0,1\}^n} (-1)^{x.z} \, |z\rangle \right) |-\rangle \\ &= \frac{1}{2^n} \sum_{z\in\{0,1\}^n} \sum_{z\in\{0,1\}^n} (-1)^{f(x)+x.z} \, |z\rangle |-\rangle \end{aligned}\right\} \quad \text{Eqn. (13.31)}$$

Analysis of the State $|\psi_4\rangle$ Before and after Measurement

- Before measurement, we can write the state $|\psi_4\rangle$ as

$$|\psi_4\rangle = (-1)^{f(x)} \sum_{z\in\{0,1\}^n} \left(\frac{1}{2^n} \sum_{x\in\{0,1\}^n} (-1)^{x.z} \right) |z\rangle|-\rangle \quad \text{Eqn. (13.32)}$$

- The amplitude of $|z\rangle = |0 \dots 0\rangle$ up to a global phase $(-1)^{f(x)}$ is

$$\frac{1}{2^n} \sum_{x\in\{0,1\}^n} (-1)^{x.0\dots0} = \frac{1}{2^n} \sum_{x\in\{0,1\}^n} (-1)^{0} = \frac{1}{2^n} \sum_{x\in\{0,1\}^n} 1 = \frac{1}{2^n} 2^n = 1 \quad \text{Eqn. (13.33)}$$

- If f is constant $[f(0) = f(1)]$, measuring the first n qubits in the standard basis will yield $|0 \dots 0\rangle$ with certainty.

Analysis of the State $|\psi_4\rangle$ Before and after Measurement

- When the function f is balanced $[f(0) \neq f(1)]$, we cannot factor out the global phase term $(-1)^{f(x)}$ as we did for the case of f being constant.

- As in our previous example for the case when $|z\rangle = |0 \ldots 0\rangle$, we can begin with

$$\frac{1}{2^n} \sum_{x \in \{0,1\}^n} (-1)^{f(x)+x.0\ldots0} = \frac{1}{2^n} \sum_{x \in \{0,1\}^n} (-1)^{f(x)} \qquad \text{Eqn. } (13.34)$$

- When f is balanced, half the terms in the sum have $f(x) = 0$ and half the terms have $f(x) = 1$; the terms in the sum cancel. The amplitude is therefore zero, which means we never see $|0 \ldots 0\rangle$ in the final measurement.

- Our observations lead us to conclude that when the measurement outcome is 0^n then output is constant; otherwise for any other n-bit measurement the output is balanced.

Summary of the Deutsch-Josza Algorithm

- Inputs to a black box U_f which performs the transformation

$$|x\rangle\,|y\rangle \rightarrow |x\rangle\,|y \oplus f(x)\rangle \text{ for } x \in \{0, \ldots 2^{n-1}\} \text{ and for } f(x) \in \{0,1\}$$

- The function $f(x)$ is either constant for al values of x or else $f(x)$ is balanced, that is it is equal to 1 for half of all possible values of x and is 0 for the other half of values of x.

- The outputs: 0 if and if f is constant; only one evaluation of U_f is performed is needed and it always succeeds.

Steps:

1. Initialize state
2. Created superpositions using Hadamard gates
3. Calculate f using U_f
4. Perform Hadamard transform
5. Measure to to obtain output

Summary

- Quantum Computation
  - What does quantum parallelism mean
- Discussed Deutsch’s problem and his circuit
  - Introduced the phase insight for simplifying quantum computational equations
- Discussed some helpful things about the Hadamard operator